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TRUSSED DIAPHRAGM IN A RIGID BENT SYSTEM

Mortimer Margolin[1]

SYNOPSIS

This paper presents a method for the determination of the reactions of a system of parallel rigid bents connected together at the roof by a continuous truss. The method pertains to a system that neglects the bracing effect of roofing, siding, purlins, and girts. This method is a close approximation which employs geometric considerations based on the premise that no change in length of the chords and verticals of the roof truss takes place.

INTRODUCTION

In Figure 1, a series of identical parallel rigid bents is shown connected by a continuous roof truss. The chords of the truss are composed of eave and ridge struts and the verticals of the truss are composed of roof beams. The bracing is composed of tension diagonals with counter diagonals. The premise of this paper, namely, the neglect of the change in length of the chords and verticals, makes simultaneous action of a diagonal and its counter impossible. All panel points are considered pinned and the bracing effect of roofing, siding, purlins, and girts is disregarded. The load P is applied at the eave level of bent 4, about which the system is symmetrical. The bent shown in Figure 1 will be used throughout this paper. It is of such a stiffness that a load of 5 kips applied at the eave level deflects the bent one inch at the eave level. By virtue of the roof truss all the bents act together to resist the load P, each bent taking a portion of the load.

In Figure 2, the first two bays are shown in a deflected position. Because of the premise in this paper the bays become a series of parallelograms. Due to symmetry, there is no rotation of the system. From the geometry shown in Figure 2, the following equations express the relationship between the deflections of the bents:

$$\text{(A)} \quad S_2 = S_1 + \sqrt{2}\, Y_a$$

but

$$S_2 = \frac{1}{5} R_2 \qquad \text{and} \qquad S_1 = \frac{1}{5} R_1$$

and

$$Y_a = \frac{\sqrt{2}\ R_1 (240\ \sqrt{2})}{\frac{1}{\sqrt{2}} \times 30 \times 10^3} = .016\ \sqrt{2}\ R_1$$

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Substituting these into Equation A, results in:

$$R_2 = R_1 + .16\,R_1 = 1.16\,R_1$$

The same reasoning applies between any two successive bents.

$$\therefore\; S_3 = S_2 + .032(R_1 + R_2)$$
$$R_3 = R_2 + .16\,(R_1 + R_2)$$
$$R_3 = 1.16\,R_1 + .16\,(1.00 + 1.16)\,R_1 = 1.50\,R_1$$

Similarly

$$S_4 = S_3 + .032\,(R_1 + R_2 + R_3)$$
$$R_4 = R_3 + .16\,(\,1.00 + 1.16 + 1.50)\,R_1$$
$$R_4 = 1.50\,R_1 + .59\,R_1 = 2.09\,R_1$$

By symmetry

$$R_1 = R_7$$
$$R_2 = R_6$$
$$R_3 = R_5$$

Summarizing the results:

$$R_2 = R_6 = 1.16\,R_1$$
$$R_3 = R_5 = 1.50\,R_1$$
$$R_4 = 2.09\,R_1$$

From these reactions a complete solution of the system can be determined.

Error Arising Out of Premise

A complete solution by energy methods, taking into account all the members of Figure 1, results in the following expression for the internal energy, U, of the system,

$$\text{(B)} \qquad 10^3\,U = 314\,R_1^2 + 270\,R_2^2 + 234\,R_3^2 + 100\,R_4^2$$
$$+ 147\,R_1\,R_2 + 68\,R_2\,R_3 + 70\,R_1\,R_3$$

Taking the partial derivatives of the internal energy with respect to redudants R_2, R_3, and R_4 and setting them equal to zero, the solution of the resulting simultaneous equations are as follows:

$$R_2 = 1.22\ R_1$$
$$R_3 = 1.63\ R_1$$
$$R_4 = 2.30\ R_1$$

If the energy of the chords and verticals is neglected, which is in accordance with the premise in this paper, the equation of energy becomes:

$$\text{(C)} \quad 10^3\ U = 296\ R_1^2 + 264\ R_2^2 + 232\ R_3^2 + 100\ R_4^2 + 128\ R_1\ R_2 + 64\ R_2\ R_3 + 64\ R_1\ R_3$$

Solving as in Equation B,

$$R_2 = 1.16\ R_1$$
$$R_3 = 1.50\ R_1$$
$$R_4 = 2.09\ R_1$$

The results of Equation C are identical with the results of the method developed in this paper. Both of these results were based on the same premise which serves to illustrate that the method developed in this paper is equivalent to an energy solution which neglects the energy of the chords and verticals.

The accuracy of the results arising out of the premise in this paper is directly proportional to some function of the area of the chords and verticals and also directly proportional to some function of the bent stiffnesses. It is inversely proportional to some function of the number of bents and to some function of the area of the diagonals. The phrase "some function" serves to emphasize that a non-linear relationship exists between the accuracy and these variables. In practice these variables are almost always in favor of yielding results which are quite accurate for engineering application. The result of Equation B compared to those of Equation C are an example of the accuracy.

Case of Unsymmetrical Loading

In Figure 3, the same system is shown as in Figure 1 except that the load is a 20 kip load applied at the midheight of the pinned column at bent 5. Also, additional diagonals, B, are placed in the lower half of the roof in the end bays. The 10 kip load at the eave level of bent 5, which prevents sway, is reversed and then distributed throughout the system. First the system is

assumed not to rotate or be subject to X-distortion. The forces required to accomplish this will be determined and then reversed and placed on the structure as shown in Figure 4. Because of identical distortion and bracing, diagonals A and B take the panel shear equally. Solving with the same reasoning as was set down in the problem of Figure 1:

$$F_2 = F_1 + \frac{1}{2}(.16\,F_1) = 1.08\,F_1$$

$$F_3 = 1.08\,F_1 + .16\,(2.08)\,F_1 = 1.41\,F_1$$

$$F_4 = 1.41\,F_1 + .16\,(3.49)\,F_1 = 1.97\,F_1$$

$$F_5 = 1.97\,F_1 + .16\,(5.46)\,F_1 = 2.84\,F_1$$

$$F_6 = F_7 + \frac{1}{2}(.16\,F_7) = 1.08\,F_7$$

$$F_5 = 1.08\,F_7 + .16\,(2.08)\,F_7 = 1.41\,F_7 = 2.84\,F_1$$

$$F_7 = 2.00\,F_1$$

$$F_6 = 2.16\,F_1$$

From $\Sigma\,Fy = 0$

$F_1 = .80^k$	$F_4 = 1.58^k$	$F_7 = 1.60^k$
$F_2 = .87^k$	$F_5 = 2.30^k$	
$F_3 = 1.12^k$	$F_6 = 1.73^k$	

From these results the forces required to prevent rotation and X-distortion of the system are determined. In Figure 4, these forces are shown acting on the structure in the opposite sense as was determined. This puts the system in rotation and introduces the longitudinal bracing reactions. It will be assumed that the longitudinal shear in the roof will be resisted by the bracing along, disregarding the shear resistance of the roof beams, inasmuch as the beams are in weak axis bending in this direction. The bracing in the longitudinal wall is also assumed to take the shear disregarding the bending resistance of the columns regardless of any fixity at the base or eave in this direction, inasmuch as the columns are in weak axis resistance in this direction. The shear (R + .4) through the lower half of the roof is divided equally between the two braces, B, due to identical distortion and bracing. The distortion is identical because of the loading and symmetry of the system. The longitudinal bracing reaction, R, is divided equally between the two wall braces, C, because of identical distortion and bracing. The upper half of the roof will be assumed subject to Y-distortion only. It is further assumed that

the reaction at bent 4 is zero and the system is symmetrical in action but reversed in direction so that:

$$f_1 = -f_7$$
$$f_2 = -f_6$$
$$f_3 = -f_5$$

All these considerations are illustrated in the geometry of Figure 5. From the assumption that the upper half of the roof is subject to Y-distortion only:

$$(D_1) \quad Y_1 = .032\,(f_1 + \tfrac{1}{2}R + .2)$$

$$(D_2) \quad Y_2 = .032\,(f_1 + f_2)$$

$$(D_3) \quad Y_3 = .032\,(f_1 + f_2 + f_3)$$

By similar triangles

$$(E) \quad Y_3 + S_3 = \tfrac{1}{2}(Y_2 + Y_3 + S_2)$$

where $S_2 = \frac{1}{5} f_2$ and $S_3 = \frac{1}{5} f_3$

Upon substitution from above

$$f_2 = 2.16\, f_3$$

Again by similar triangles

$$(G) \quad Y_3 + S_3 = \tfrac{1}{3}(Y_1 + Y_2 + Y_3 + S_1)$$

where

$$S_1 = \tfrac{1}{5} f_1$$

Upon substitution from above

$$(H) \quad .24\, f_3 = .068\, f_1 + .0053\, R + .0021$$

From congruent triangles

$$S_R + S_h = Y_3 + S_3$$

where

$S_R = .016\ R$

$S_b = .032\ (\frac{1}{2}\ R + .2)\ + \ Y_1$

Substitution from above gives:

(J) $\quad .3\ f_3 = .048\ R + .0128$

From statics and from $f_2 = 2.16\ f_3$,

(K) $\quad 6\ f_1 + 10.8\ f_3 + 2\ R = 4.7$

Solving Equations (H), (J), and (K) simultaneously

$f_1 = .38^k = -f_7$

$f_2 = .27^k = -f_6$

$f_3 = .125^k = -f_5$

$R = .53^k$

The final reactions become:

$R_1 = F_1 - f_1 = .80^k - .38^k = .42^k$

$R_2 = F_2 - f_2 = .87 - .27 = .60^k$

$R_3 = F_3 - f_3 = 1.12^k - .125^k = .995^k$

$R_4 = F_4 + 0 = 1.58^k + 0 = 1.58^k$

$R_5 = F_5 + f_5 = 2.30^k + .125^k = 2.425^k$

$R_6 = F_6 + f_6 = 1.73^k + .27^k = 2.00^k$

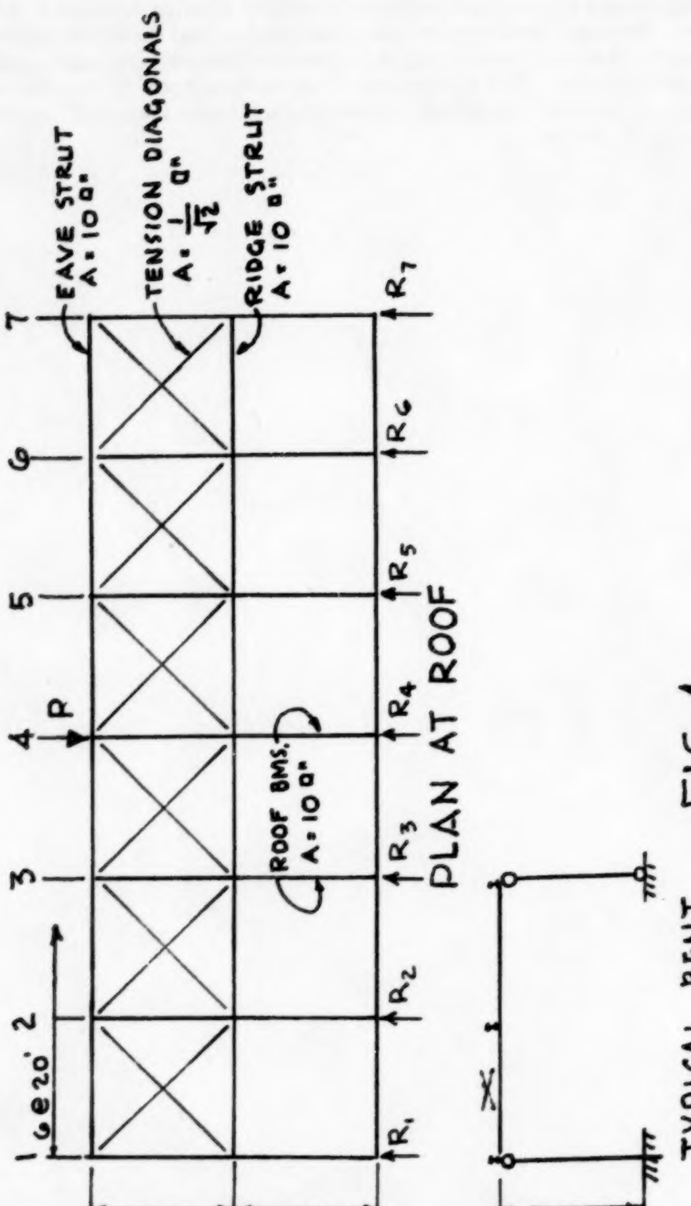
$R_7 = F_7 + f_7 = 1.60^k + .38^k = 1.98^k$

The final reactions at the bents, stresses in the roof truss and conditions at bent 5 are illustrated in Figure 6.

CONCLUSION

The method presented in this paper is an approximation based on the premise that no change in length of the chords and verticals of a roof truss diaphragm takes place. It is assumed that all shears in the roof and longitudinal walls are taken by bracing only, disregarding any resistance provided by roofing, siding, girts and purlins. In the case of unsymmetrical loading, the above premise and assumptions apply, but in addition, it is assumed that

no X-distortion in the continuous roof takes place. In the examples set forth in this paper the systems were extremely simple, but any extensions or special arrangements can be approached in a fashion similar to that set down in this paper. Systems symmetrical along the longitudinal direction were used in this paper. Any unsymmetrical case, construction-wise, would require special consideration. The advantages in the method are: (1) no necessity for the use of calculus, (2) simpler equations and calculations, (3) an intuitive approach for analysis.



EAVE STRUT
A = 10 □"
TENSION DIAGONALS
A = $\frac{1}{\sqrt{2}}$ □"
RIDGE STRUT
A = 10 □"
ROOF BMS.
A = 10 □"
P
1 2 3 4 5 6 7
6 @ 20'
R_1 R_2 R_3 R_4 R_5 R_6 R_7
20'-0"
20'-0"
20'-0"
PLAN AT ROOF
TYPICAL BENT

FIG. 1

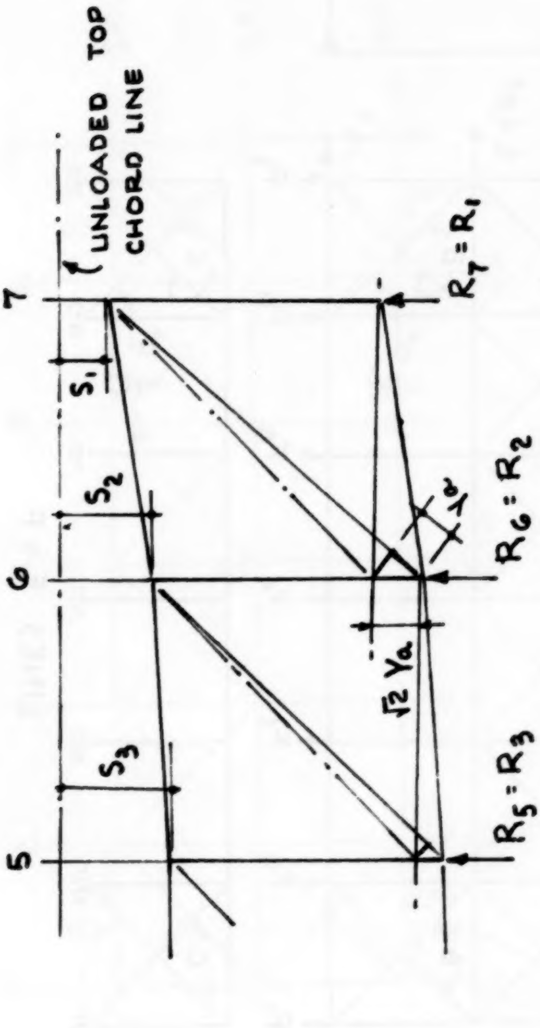
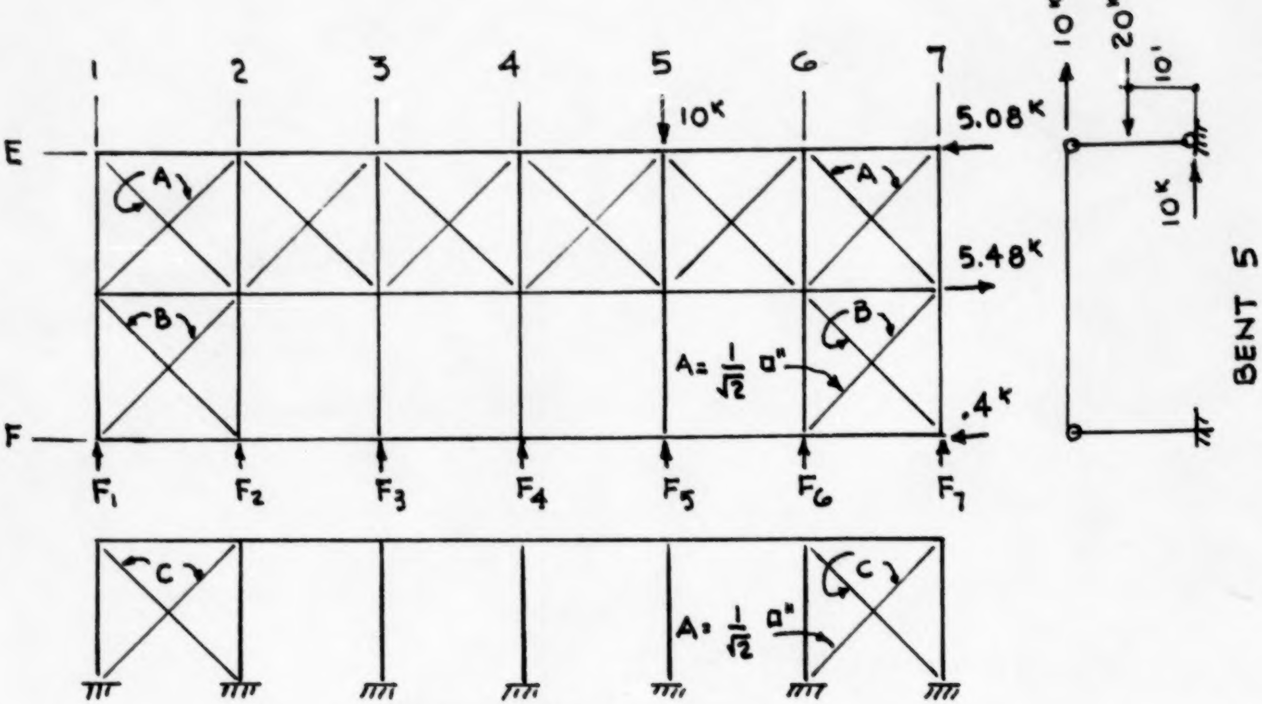


FIG.2



LINES E & F

FIG. 3

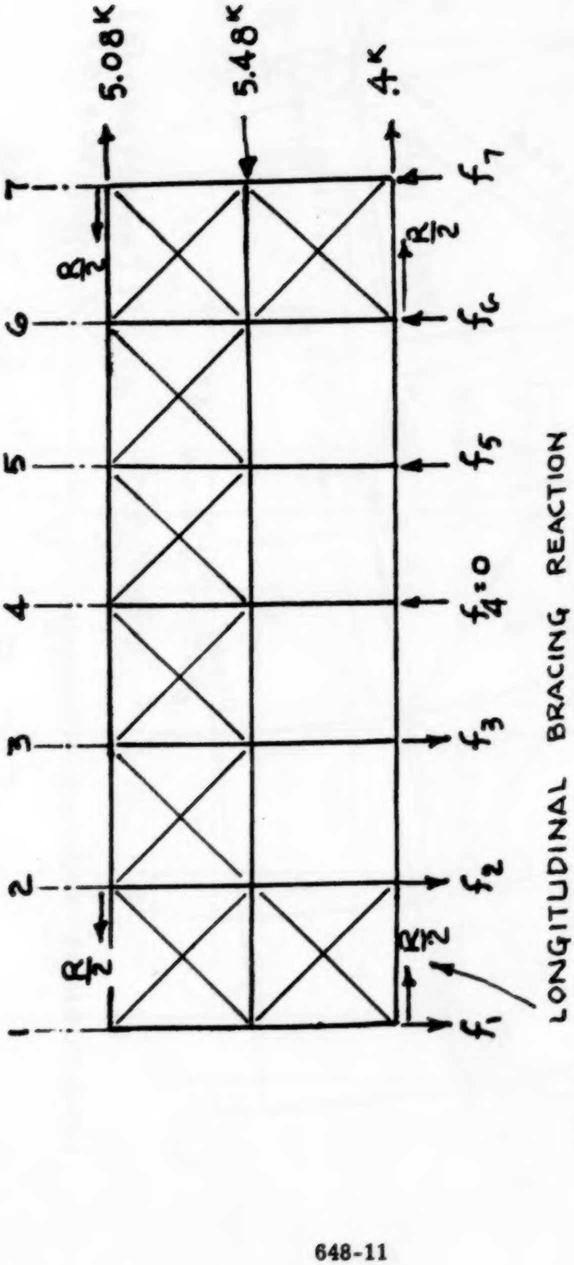
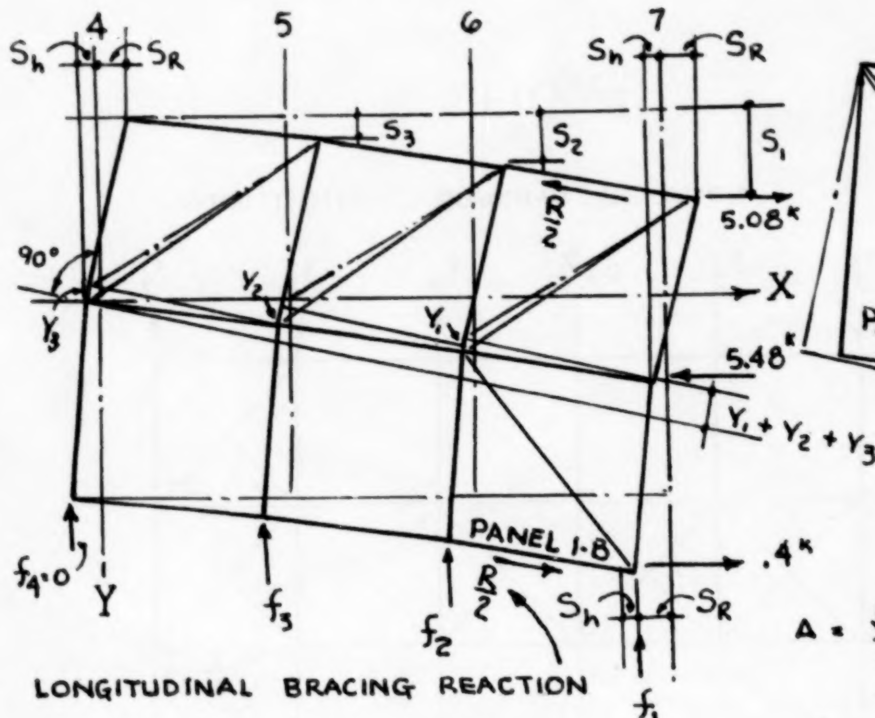
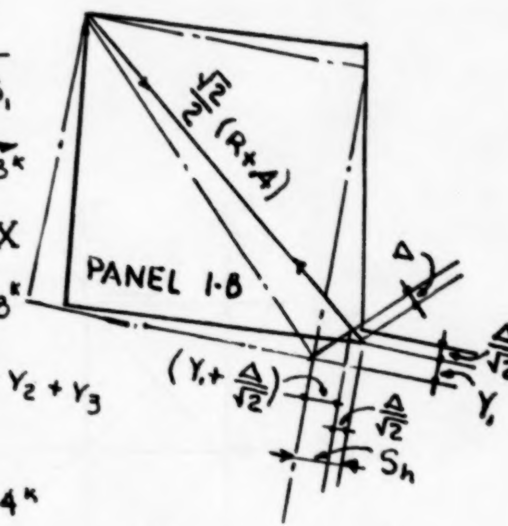

1
2
3
4
5
6
7
5.08K
5.48K
4K
R/2
R/2
R/2
R/2
f1
f2
f3
f4=0
f5
f6
f7
LONGITUDINAL BRACING REACTION

FIG. 4.





$$\Delta = \frac{\sqrt{2}\,(R+.4)\,(240\sqrt{2})}{2 \times \frac{1}{\sqrt{2}} \times 30 \times 10^3}$$

$$S_h = Y_1 + \frac{2\Delta}{\sqrt{2}}$$

FIG. 5

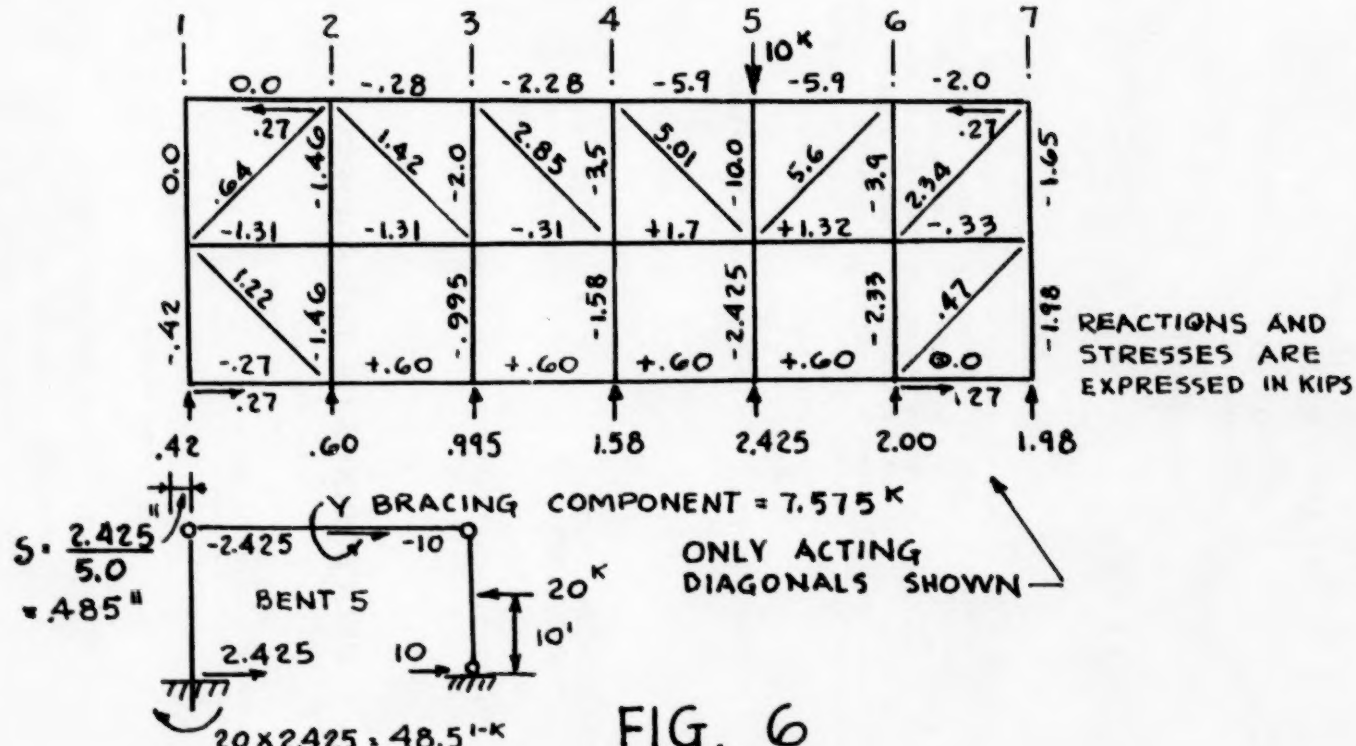

REACTIONS AND STRESSES ARE EXPRESSED IN KIPS
Y BRACING COMPONENT = 7.575 K
ONLY ACTING DIAGONALS SHOWN
BENT 5
S = 2.425 / 5.0 = .485"
20 X 2.425 = 48.5 I-K
10K
20K
10'

FIG. 6